A Comparison of Stochastic, Robust and Distributionally Robust Model for Earthquake Shelter Location-Allocation Problem

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Abstract: Decide where to locate shelters and how to allocate evacuees to each shelter are of great importance to disaster management. Due to the existence of uncertainty, it is quite challenging to make such decisions while considering unpredictable demand. Traditional stochastic or robust optimization model is either too progressive or conservative. In this research, we propose a distributionally robust optimization (DRO) model, in which uncertain evacuation demand are described as a moment-based ambiguous set. We compare stochastic, robust and distributionally robust optimization model in the case of Tokyo using sample data derived from history earthquake damage data. The results show that the total cost in DRO model is between which in stochastic and robust optimization model.

Keywords: shelter, earthquake, distributionally robust optimization, stochastic optimization, robust optimization

1. Introduction

Earthquake has always been a major threaten to Japan, which caused huge loss in recent years. It is predicted that there may be an inland earthquake hitting Tokyo in near 30 years. To mitigate the possible losing of life and property, it is of great importance to ensure that all evacuees can get to shelters in the shortest possible time. In Tokyo metropolitan, the usual practice is to designate public elementary school, public middle school and other public facilities (high school, gym, etc.) as shelters. By the disaster prevention plan, these shelters are capable of accepting most evacuees under emergency conditions. However, in the context of novel coronavirus, shelters cannot accommodate the same number of evacuees as before due to the necessary distance needed to avoid possible exposure. Therefore, more shelters are in need. The challenge lies in selecting the best option from multiple potential shelters. Moreover, the demand, which means the number of people who need going to shelter, remains uncertain before making the choice. These difficulties make the problem of shelter location selection hard to solve.

To cope with demand uncertainty in shelter location problem, previous studies mainly adopt stochastic programming (SP) (A. C. Y. Li et al., 2012; L. Li et al., 2011; Xu et al., 2018) and robust optimization (RO) (Kulshrestha et al., 2011). SP minimizes the expected value of objective function with the assumption that the probability distribution of scenarios is known, while RO only consider the worst scenario. These two approaches are either too progressive or too conservative. In this study, we consider a two-stage distributionally robust model, in which we make decision that makes the expected value for the worst-case probability distribution within a set of distributions (Bansal et al., 2018). We expect that DRO will make appropriate decisions to better help authority decide the location of shelters.

2. Literature review

This part will review the literature on: (i) SP and RO model for location-allocation problem; (ii) DRO model.

2.1. SP and RO studies for location-allocation problem

Shelter location-allocation problem has been widely discussed in disaster management literature. Some works

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do not specifically study on shelters, but they also discuss similar location choice problem in disaster (Dönmez et al., 2021; Grass & Fischer, 2016). Earthquake (Balcik & Beamon, 2008; Döyen et al., 2012; Mete & Zabinsky, 2010; Xu et al., 2018), hurricane (A. C. Y. Li et al., 2012; L. Li et al., 2011; Rawls & Turnquist, 2011) and flood (Chang et al., 2007; Maqsood & Huang, 2013) are most concerned disaster type. Since disaster management can be roughly classified into pre- and post-disaster phase, two-stage paradigm is widely used. In the pre-disaster stage (preparedness stage), decisions are often made on location of facility (A. C. Y. Li et al., 2011; Mete & Zabinsky, 2010; Ozbay et al., 2019), capacity (Cai et al., 2011; A. C. Y. Li et al., 2011) or pre-positioned relief (Davis et al., 2013; Rawls & Turnquist, 2011). Minimization of costs is most commonly used for objective function in first stage. These costs include the fixed cost for a new shelter (lighting equipment, electric generator, toilet, etc.), variable cost corresponding to one evacuee (food, water, blanket, etc.). The objective of first stage is to minimize the fixed or variable cost of selected facilities, the cost of pre-positioned items. Evacuation flow, unmet demand (A. C. Y. Li et al., 2011; L. Li et al., 2011), relief flow, shortage and unused relief (Alem et al., 2016; S. L. Hu et al., 2015) are usually decision variables of the post-disaster stage (response stage). Objective of the second stage is often to minimize the transportation cost, penalty cost for unmet demand, shortage and surplus items. The penalty cost is set by authorities to 'punish' the waste of relief, excess or insufficient capacity in order to make the selected shelters just meet the demand.

In the second stage, uncertainties are revealed, therefore, capturing uncertainties of disaster is the main challenge. Among different uncertainties (supply uncertainty, demand uncertainty, network connectivity uncertainty) considered in location-allocation problem, demand uncertainty is the most common, where a large body of literature focus on (L. Li et al., 2011; Mostajabdaveh et al., 2019; Ozbay et al., 2019). Stochastic programming (SP) and robust optimization (RO) are both common paradigms to deal with uncertainties in location-allocation problems. When the probability distribution of random scenarios is known or can be assumed to follow certain distribution, SP, which minimizes the expected value of second stage objective function with a predefined probability distribution can be used. The method to solve such stochastic programming can be divided into two groups by the type of solution. The first group is exact solution method. The most basic algorithm is L-shaped method (Slyke, R M Van; Wets, 1969), which approximates the nonlinear term in the objective of problems by adding cutting plane. In original L-shaped algorithm, the decision variable in first and second stage are continuous variable. However, it has been proved that if there are only pure integer variables in first stage, the optimal solution can be obtained in finite iterations (Ahmed et al., 2004; Laporte et al., 1994). Moreover, if there exists both continuous and integer variables (i.e., mixed integer), it may need more cutting plane before finally convergence (Sanci & Daskin, 2021). However, if integer or mixed integer variables are in second stage, these optimality cuts can no longer be obtained by LP duality which is used in classical L-shaped algorithm. Most location-allocation problems do not involve integer variable in second stage, therefore, L-shaped algorithm has good application if the problem size is not too large (L. Li et al., 2011; Liu et al., 2009; Noyan et al., 2016). In real-world, there are many options and scenarios, which could make the problem size quite large. In such cases, heuristics can be employed to get an approximate solution in a short time (F. Hu et al., 2014; A. C. Y. Li et al., 2011; Yu et al., 2018). If the distribution is unknown and cannot be assumed to match certain distribution, robust optimization is a good way to model uncertainty (Kulshrestha et al., 2011). In DRO model, an uncertainty set is assumed for each random parameter, the decision is made to ensure that the solution is feasible in whatever value of the parameter.

2.2. Distributionally robust optimization

Distributionally robust optimization (DRO) can be seen as a combination of SP and RO. Unlike SP and RO, DRO aims to optimize the objective function value under worst probability distribution. DRO constructs an ambiguity set which includes different probability distributions, and the solution should be feasible in any distribution within the set. In previous studies, ambiguity sets can be categorized into two groups: moment-based and discrepancy-based ambiguity sets (Rahimian & Mehrotra, 2019). Typically, a moment-based ambiguity set sets the lower and upper bound of moments of possible probability distributions, while discrepancy-based ambiguity sets contain the distributions which are close to certain distribution. There are also some special ambiguity sets, like Markov ambiguity set, Chebyshev ambiguity set, Gauss ambiguity set, Median-absolute deviation ambiguity set, Huber ambiguity set, Hoeffding ambiguity set, Bernstein ambiguity set, Choquet ambiguity set, Mixture ambiguity set. DRO can be reformulated as semi-infinite programming (SIP), which can be solved by cutting-surface method (Bansal et al., 2018; Rahimian et al., 2019) or dual method (Bertsimas et al., 2010; Delage & Ye, 2010; Wiesemann et al., 2013). Compared to SP and RO, DRO shows the merit of great degree of freedom to construct the uncertainty set, also, in real-world problem, the objective value calculated by DRO model will not be too high or too low, which makes it suitable to solve location-allocation problem. Up to now, although there is a large body of literatures adopt SP or RO to solve shelter location-allocation problem, it seems that DRO has not been used in such problem.

3. Model formulation and solution method

In this study, we employ a two-stage paradigm to model demand uncertainty and decision-making. In the first stage (preparedness stage), the authority should decide the location and capacities of new shelters. In the second stage, the authority needs to allocate evacuees from each demand point to each shelter (including existing shelters and new shelters). The number of evacuees in each demand point after earthquake is the uncertain parameter, which can be got from history data or simulation results. We apply SP, RO, DRO respectively to model the uncertainty and compare the results.

These three models share the same parameters and decision variables as follows:

Table 1 model parameter

First stage	f_i	Fixed cost for shelter i, including necessary equipment (lighting, power, toilet, etc.) for opening a shelter
	v	Variable cost, referring to the relief cost (food, water, blanket, mask, other daily necessities, etc.) corresponding to one evacuee
	PS	Set of potential shelters
Second stage	d_{ki}	Transportation cost (vehicle, fuel, etc.) of allocating one evacuee from demand point k to shelter i, which is in proportion to the distance between k and i
	m	Unit penalty cost for unmet demand
	n	Unit penalty cost for unused capacity
	P_s	Probability distribution of scenarios
	$D_k(s)$	Demand at demand point k in scenario s
	ES	Set of existing shelters
	K	Set of demand points
	S	Set of scenarios
	U	moment-based ambiguity set

Table 2 model decision variable

First stage	x_i	1 if potential evacuation center i is selected, 0 otherwise
	c_i	Capacity of evacuation center i
Second	$q_{ki}(s)$	Number of evacuees being

stage		allocated from demand point k to selected shelter i
	$q_{kj}(s)$	Number of evacuees being allocated from demand point k to existing shelter j
	$z_k^-(s)$	Unmet demand at demand point k
	$z_i^+(s)$	Unused capacity at selected shelter i
	$z_j^+(s)$	Unused capacity at existing shelter j

The objective function (1) for first stage is to minimize the fixed cost of new shelters and the variable cost which corresponding to capacity. $\pi(\mathrm{x,c,s})$ is the objective function value of second stage given the value of x and c in scenario s. Equation (1.1) is SP, where $E_s[\pi(x,c,s)]$ is the expected value of $\pi(\mathrm{x,c,s})$, in this study we assume that each scenario has equal probability. Equation (1.2) is RO, where $\max_{s\in S}[\pi(x,c,s)]$ is the worst-case by selecting the scenario which makes the cost of second stage max. Equation (1.3) is DRO, where $\max_{P_s\in U} E_s[\pi(x,c,s)]$ is the worst-case by selecting the probability distribution which makes the cost of second stage max. Constraint (4), where M is a big number, ensures that the capacities of unselected shelters are 0.

$$\min \sum_{i\in PS} f_i x_i + \sum_{i\in PS} c_i v + Q \tag{1}$$

$$\mathrm{Q} = E_s[\pi(x,c,s)] \tag{1.1}$$

$$\mathrm{Q} = \max_{s\in S}[\pi(x,c,s)] \tag{1.2}$$

$$\mathrm{Q} = \max_{P_s\in U} E_s[\pi(x,c,s)] \tag{1.3}$$

s.t.

$$x_i \in \{0,1\} \tag{2}$$

$$c_i \geq 0 \tag{3}$$

$$c_i \leq M x_i \tag{4}$$

In the second stage, the objective function (5) minimizes the cost of allocating people from demand points to shelters, the penalty cost of unused capacity and unmet demand. Constraints (6) and (7) are used to get the possible unused capacity of each shelter (number of evacuees from all demand points should not exceed the capacities of shelters). Constraint (8) is to obtain the possible unmet demand in each demand point.

$$\pi(\mathrm{x,c,s}) = \min \left\{ \begin{array}{c} \sum_{k\in K}\sum_{i\in PS} d_{ki} q_{ki}(s) + \\ \sum_{k\in K}\sum_{j\in ES} d_{kj} q_{kj}(s) + \\ m\sum_{k\in K} z_k^-(s) + n\sum_{i\in PS} z_i^+(s) + \\ n\sum_{j\in ES} z_j^+(s) \end{array} \right\} \tag{5}$$

s.t.

$$\textstyle\sum_{k\in K} q_{kj}(s) - c_j + z_j^+(s) = 0 \quad \forall j \in ES \tag{6}$$

$$\sum_{k\in K} q_{ki}(s) - c_i + z_i^+(s) = 0 \quad \forall i \in PS \tag{7}$$

$$\sum_{i\in PS} q_{ki}(s) + \sum_{j\in ES} q_{kj}(s) + z_k^-(s) - D_k(s) = 0 \quad \forall k \in K \tag{8}$$

$$q_{ki}(s), q_{kj}(s), z_j^+(s), z_i^+(s), z_k^-(s) \geq 0 \tag{9}$$

Here we use the classical L-shaped algorithm to solve SP, RO, and use a modified L-shaped algorithm to solve DRO. First, we write the dual problem of $\pi(\mathrm{x,c,s})$ as (10)-(15), then rewrite the original two-stage problem into the so-called master problem as (1), (16), (2), (3), where λ, ρ is the decision variable of the sub dual problem.

$$\overline{\pi}(\mathrm{x,c,s}) = \max\left\{-\sum_{i\in PS,ES} c_i\lambda_i - \sum_{k\in K} D_k(s)\rho_k\right\} \tag{10}$$

$$\rho_k + \lambda_i + d_{ki} \geq 0 \quad \forall k \in K \;\; i \in PS \tag{11}$$

$$\rho_k + \lambda_j + d_{kj} \geq 0 \quad \forall k \in K \;\; j \in ES \tag{12}$$

$$\lambda_i \geq -n \quad i \in PS \tag{13}$$

$$\lambda_j \geq -n \quad j \in ES \tag{14}$$

$$\rho_k \geq -m \quad k \in K \tag{15}$$

$$\pi(x, c, s) \geq -\sum_{i \in PS} c_i \lambda_i(s) - \sum_{j \in ES} c_j \lambda_j(s) - \sum_{k \in K} D_k(s)\, \rho_k \tag{16}$$

The algorithm is as follows:

***Step* 0:** Set iteration $l = 0$, lower bound $LB = -\infty$, upper bound $UB = \infty$, initial optimal value x^1, c^1.

***Step* 1:** For $s \in S$, solve sub dual problem, get optimal solution $\lambda^l(s)$, $\rho^l(s)$ and optimal objective value $\pi(x^l, c^l, s)$.

***Step* 2:** For RO, set the probability of worst-case as 1, other scenarios 0. For SP, set equal probability to each scenario. For DRO model, solve distribution separation problem (17)-(19) to get probability distribution P_S. Distribution separation problem (Bansal et al., 2018) is used to get the probability distribution from a moment-based uncertainty set, lb, ub is the lower and upper bound for first moment of uncertain demand set, $\overline{lb}$ and $\overline{ub}$ is the lower and upper bound for second moment of uncertain demand set, p_s^l is the probability of scenario s in iteration l:

$$\max \sum_{s \in S} p_s^l\, \pi(x^l, c^l, s) \tag{17}$$

$$lb_k \leq \sum_{s \in S} p_s^l\, D_k(s) \leq ub_k \quad \forall k \in K \tag{18}$$

$$\overline{lb}_k \leq \sum_{s \in S} p_s^l \big(D_k(s) - E_s(D_k)\big)^2 \leq \overline{ub}_k \quad \forall k \in K \tag{19}$$

***Step* 3:** If $UB > \sum_{i \in PS} f_i x_i^l + \sum_{i \in PS} c_i^l v + Q$, update UB as the right-hand side value, if $UB \leq LB + \varepsilon$, iteration stops, ε is the tolerance gap.

***Step* 4:** Solve master problem (1)-(4), (10) by adding optimality cut got in step 1:

$Q \geq E_s[-\sum_{i \in PS} c_i \lambda_i^l(s) - \sum_{j \in ES} c_j \lambda_j^l(s) - \sum_{k \in K} D_k(s)\, \rho_k^l(s)]$,

where E_s means the expected value following the probability distribution got in step 2. In this step we get optimal value x^l, c^l, update LB if the objective value of master problem is bigger than LB. $l = l + 1$. Return to step 1.

4. Case study in Meguro City

To test the algorithm and compare the results of three models, a case study was conducted in Meguro City.

4.1. Case description and data

Meguro City is located in the southwest of Tokyo Metropolitan, with 278,654 residents by the end of 2021. This case consisted of 88 demand points, 38 existing shelters and 38 potential shelters. Almost all the existing shelters are public school. We estimated the capacities of these shelters by effective sheltering area and per capital sheltering area. We set the per capital sheltering area as 6.6 m^2, which is double of the area before the spread of novel coronavirus. Therefore, the estimated capacity of school can be obtained by equation (20), where 0.7 is the ratio of effective area to total area.

The uncertain demand set was got by the following steps: (i) get the population in each building by assuming that the population is proportional to building area; (ii) get the attribute (age and structure) of each building (building structure was known, then we only need to randomly generate the building age); (iii) calculate the completely collapse rate for each building by equation (21) using the parameters of normal distribution shown in table 3, where $P(PGV)$ (PGV is peak ground velocity, and we set it as 150 cm/s) is the completely collapse probability, λ and ζ is expectation and standard deviation; (iv) randomly generate the state of each building (collapse or not) by the probability we got in step (iii). By these four steps we can get one scenario, then we repeated the process for 100 times, then we got 100 scenarios.

$$capacity = \left(area_{gym} + area_{classroom}\right)\frac{0.7}{3.3 \times 2} \quad (20)$$

$$P(PGV) = \Phi\left(\frac{\ln(PGV) - \lambda}{\zeta}\right) \quad (21)$$

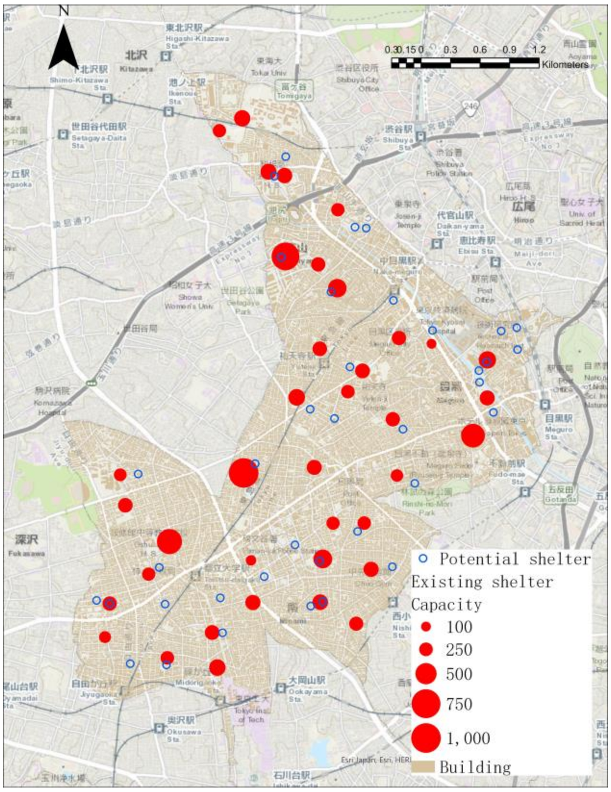

Figure 1 existing shelter and potential shelter in study area (Meguro City)

Table 3 parameters for different structure and age

Building structure	Building age	λ	ζ
Wooden	~ 1971	4.84	0.71
	1972 ~ 1981	5.11	0.76
	1982 ~ 1991	5.41	0.64
	1992 ~ 2001	5.70	0.70
	2002 ~	6.62	0.89
RC	~ 1971	5.12	0.65
	1972 ~ 1981	5.33	0.58
	1982 ~	6.00	0.79
Steel	~ 1971	4.64	0.62
	1972 ~ 1981	4.97	0.49
	1982 ~	5.64	0.73

4.2. Results and analysis

Since the penalty cost can be subjectively decided by authorities, it reflects the preference for conservative or radical decision-making style. For example, if the ratio of n (unit penalty cost for unused capacity) to m (unit penalty cost for unmet demand) is bigger than 1, it means that compared to possible waste of capacity, the authority cares more about the number of evacuees who could use shelters. Moreover, if the ratio is very large, it shows that the authority hopes to ensure all evacuees could go to shelter no matter how many capacities may be wasted. We set the ratio of n to m as 1, meaning that we do not want either excess or insufficient capacity to the same degree, and apply these parameters and uncertainty demand set into SP, RO, DRO respectively, then we got the results of selected shelters and their capacities as in figure 2. The locations being selected and capacities being decided are totally different in three models. RO selected 12 shelters with a total capacity of 8,773, which are both the most among three models. This is because in RO model, the demand in each demand point is equal or greater than which is in SP and DRO model, therefore, more shelters should be chosen to ensure all the demand points meet their demands. DRO selected 6 shelters with capacity of 6,593, and SP selected 10 shelters with capacity of 8,618. Although the cost of DRO in first stage is smaller than SP, the total cost of DRO, which includes transportation cost and penalty cost, is bigger than SP. Compared to SP which minimizes the expected value of second stage objective function, DRO minimizes the maximum value of second stage objective function among a set of probability distributions (ambiguity set). It is expected that the second stage objective value of DRO should be bigger than SP. It indicates that for this proposed DRO model, we can save costs in the first stage while ensure as many evacuees go to shelters as possible in the meantime.

Change the ratio of n to m, and run these three models to get the total cost. We can see from figure 3 that: (i) the total cost of DRO is between RO and SP; (ii) the cost of RO is much higher than SP and DRO; (iii) cost of DRO is just a litter higher than SP, and as the ratio of n to m rises, the gap seems to be enlarged. This corresponds to our assumption, since DRO optimizes the objective function under the worst probability distribution, therefore, its value must be equal or bigger than SP which optimizes under a pre-defined distribution. RO gets the biggest objective value because it must ensure feasibility under the worst scenario. In our case it means that the shelters selected by DRO should always meet the biggest demand.

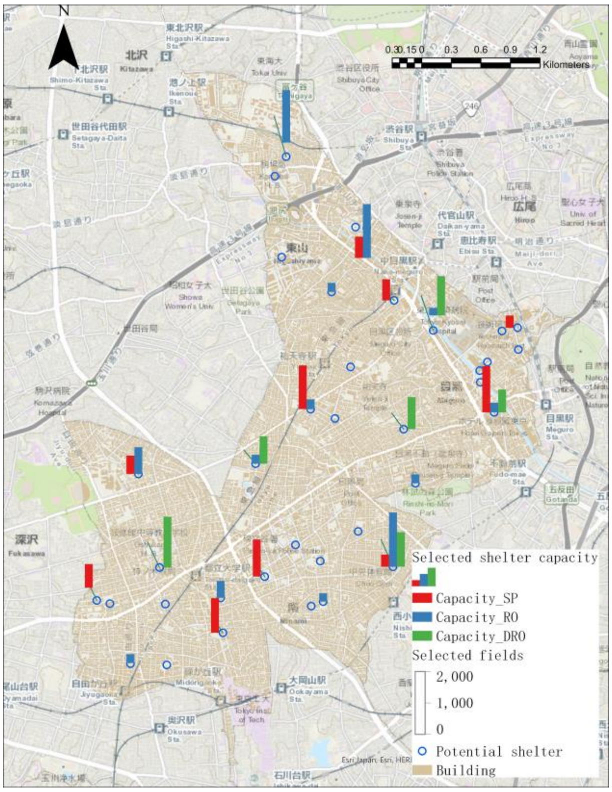

Figure 2 selected shelters and capacities for $n{:}m = 1$

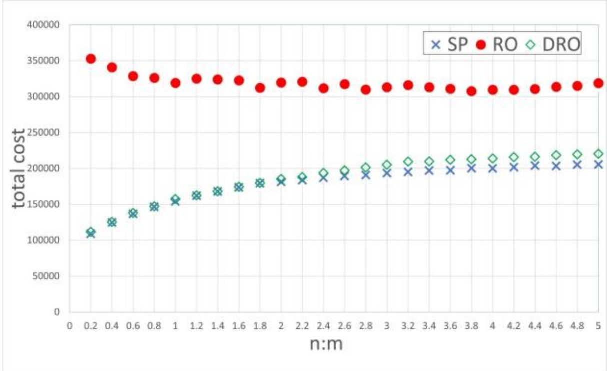

Figure 3 total cost of SP, RO and DRO

5. Discussions and conclusions

This study adopts distributionally robust optimization (DRO) in shelter location-allocation problem by constructing a moment-based ambiguity set. To the best of our knowledge, it is the first time to incorporate DRO to such problem. A case study in Meguro City was conducted to compare the objective value of SP, RO and DRO. It shows that the value of DRO is between another two models. This proposes a new way to solve shelter location-allocation problem which can save cost under a limit budget compared to conservative RO model. Also, it ensures that more evacuees can use shelters compared to SP model, which is relatively reliable under extreme conditions. However, there are still several points that need further study in the future: (i) we only use moment-based ambiguity set, in fact there are many other sets available; (ii) the gap of DRO and SP seems to increase as parameter changes, how these parameters works on the cost gap remains unknown; (iii) the algorithm need to be improved to be more effective.

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